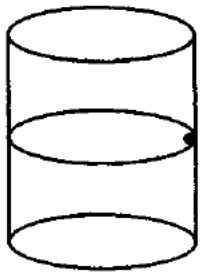
Take $g = 9{\cdot}8$ ms^{-2} and give all answers correct to 3 significant figures where necessary.

1. A motorcyclist rides in a cylindrical well of radius 5 m. He maintains a horizontal circular path at a constant speed of 10 ms^{-1}. The coefficient of friction between the wall and the wheels of the cycle is μ.



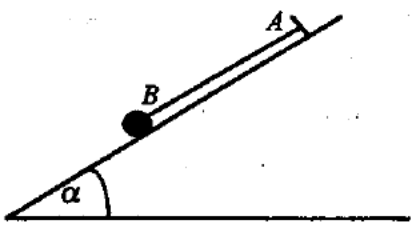
Modelling the cyclist and his machine as a particle in contact with the wall, show that he will not slip downwards provided that $\mu \geq 0{\cdot}49$. **(7 marks)**

2. A particle P moves with simple harmonic motion in a straight line. The centre of oscillation is O. When P is at a distance 1 m from O, its speed is 8 ms^{-1}. When it is at a distance 2 m from O, its speed is 4 ms^{-1}.

(a) Find the amplitude of the motion. **(4 marks)**

(b) Show that the period of motion is $\frac{\pi}{2}$ s. **(3 marks)**

3. A particle of mass m kg is attached to the end B of a light elastic string AB. The string has natural length l m and modulus of elasticity λ. N.



The end A is attached to a fixed point on a smooth plane inclined at an angle α to the horizontal, as shown, and the particle rests in equilibrium with the length $AB = \frac{5l}{4}$ m.

(a) Show that $\lambda = 4\,mg \sin \alpha$. **(3 marks)**

The particle is now moved and held at rest at A with the string slack. It is then gently released so that it moves down the plane along a line of greatest slope.

(b) Find the greatest distance from A that the particle reaches down the plane. **(6 marks)**

4. The acceleration a ms^{-2} of a particle P moving in a straight line away from a fixed point O is given by $a = \frac{k}{1+t}$, where t s is the time that has elapsed since P left O, and k is a constant.

(a) By solving a suitable differential equation, find an expression for the velocity v ms^{-1} of P in terms of t, k and another constant c. **(3 marks)**

Given that $v = 0$ when $t = 0$ and that $v = 4$ when $t = 2$,

(b) show that $v \ln 3 = 4 \ln (1 + t)$. **(3 marks)**

(c) Calculate the time when P has a speed of 8 ms^{-1}. **(3 marks)**

5. A particle of mass m kg, at a distance x m from the centre of the Earth, experiences a force of magnitude $\frac{km}{x^2}$ N towards the centre of the Earth, where k is a constant. Given that the radius of the Earth is $6{\cdot}37 \times 10^6$ m, and that a 3 kg mass experiences a force of 30 N at the surface of the Earth,

(a) calculate the value of k, stating the units of your answer. **(3 marks)**

The 3 kg mass falls from rest at a distance $x = 12{\cdot}74 \times 10^6$ m from the centre of the Earth. Ignoring air resistance,

(b) show that it reaches the surface of the Earth with speed $7{\cdot}98 \times 10^3$ ms^{-1}. **(7 marks)**

In a simplified model, the particle is assumed to fall with a constant acceleration 10 ms^{-2}. According to this model it attains the same speed as in (b), $7{\cdot}98 \times 10^3$ ms^{-1}, at a distance $(12{\cdot}74 - d) \times 10^6$ m from the centre of the Earth.

(c) Find the value of d. **(3 marks)**

6. A particle P of mass 0·4 kg hangs by a light, inextensible string of length 20 cm whose other end is attached to a fixed point O. It is given a horizontal velocity of 1·4 ms^{-1} so that it begins to move in a vertical circle. If in the ensuing motion the string makes an angle of θ with the downward vertical through O, show that

(a) θ cannot exceed 60°, **(6 marks)**

(b) the tension, T N, in the string is given by $T = 3{\cdot}92(3\cos\theta - 1)$. **(4 marks)**

If the string breaks when $\cos\theta = \frac{3}{5}$ and P is ascending,

(c) find the greatest height reached by P above the initial point of projection. **(5 marks)**

7. A uniform solid sphere, of radius a, is divided into two sections by a plane at a distance $\frac{a}{2}$ from the centre and parallel to a diameter.

(a) Show that the centre of gravity of the smaller cap from its plane face is $\frac{7a}{40}$. **(9 marks)**

This smaller cap is now placed on an inclined plane whose angle of inclination to the horizontal is θ. The plane is rough enough to prevent slipping and the cap rests with its curved surface in contact with the plane.

(b) If the maximum value of θ for which this is possible without the cap turning over is 30°, find the corresponding maximum inclination of the axis of symmetry of the cap to the vertical. **(6 marks)**

